analysis of the gain of magnetostatic amplifiers employing a composite layered structure of semiconductors and ferrites [12].

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Different Representations of Dyadic Green's Functions for a Rectangular Cavity

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***Abstract*—Several different but equivalent expressions of the dyadic Green's functions for a rectangular cavity have been derived. The mathematical relations between the dyadic Green's function of the vector potential type and that of the electric type are shown in detail. This work supplements the one by Morse and Feshbach [1].**

I. INTRODUCTION

THE dyadic Green's function for a rectangular cavity has previously been studied by Morse and Feshbach [1]. The function which they introduced is of the vector potential type, hereby denoted by $\bar{G}_A$, corresponding to the dyadic version of the vector Green's function for the vector Helmholtz equation. Two forms of $\bar{G}_A$ were obtained by these authors. While one form is complete, the other one is not. These authors mentioned that the two forms are equivalent when a longitudinal part is added to the incompleted form, but the exact relations were not discussed.

In a recent paper, Rahmat-Samii [7] presented the dyadic Green's function of the electric type for rectangular waveguides and cavities, and introduced an auxiliary dyadic $\bar{g}_m$. This dyadic, however, is the dyadic Green's function of the vector potential type $\bar{G}_A$, as can easily be seen by comparing (1) in the present work with [7, eq. (9)]. As a result, [7, eq. (26)] for $\bar{g}_m$ is the same as our expression (10). The representation of the dyadic Green's functions for rectangular waveguides which is given in Rahmat-Samii's paper has previously been presented in [3] and for rectangular cavities in [6].

In this paper, we give a detailed derivation of several alternative representations of the dyadic Green's functions of both the vector potential type and the electric type for a rectangular cavity. Although the two types of functions are intimately related, it is more direct to use the function of the electric type that would bypass the tedious differentiation of discontinuous series for the evaluation of the fields in a source region.

II. DYADIC GREEN'S FUNCTIONS OF THE VECTOR POTENTIAL TYPE AND OF THE ELECTRIC TYPE

The classification of dyadic Green's functions of various types and kinds has previously been discussed [2], [3]. For the present work, it is sufficient to review two types of functions pertaining, respectively, to the vector potential function and the electric field. The dyadic Green's function

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of the vector potential type satisfies the differential equation

$$\nabla^2 \bar{G}_A + k^2 \bar{G}_A = -I\delta(\bar{R} - \bar{R}') \tag{1}$$

where

k^2 $= \omega^2 \mu_0 \varepsilon_0$;
I the idem factor $= \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}$;
$\delta(\bar{R} - \bar{R}')$ the three-dimensional δ-function $= \delta(x - x')\delta(y - y')\delta(z - z')$.

A cap is used here to denote a unit vector. The dyadic Green's function of the electric type satisfies the differential equation

$$\nabla \times \nabla \times \bar{G}_e - k^2 \bar{G}_e = I\delta(\bar{R} - \bar{R}'). \tag{2}$$

The relation between $\bar{G}_A$ and $\bar{G}_e$ is

$$\bar{G}_e = \left(I + \frac{1}{k^2}\nabla\nabla\right) \cdot \bar{G}_A. \tag{3}$$

For cavities we are seeking the kind of functions which satisfy the boundary condition

$$\hat{n} \times \bar{G}_e = 0 \tag{4}$$

or

$$\hat{n} \times \left(I + \frac{1}{k^2}\nabla\nabla\right) \cdot \bar{G}_A = 0. \tag{5}$$

Previously, we called these kinds of functions the functions of the first kind and denoted them, respectively, by $\bar{G}_{e1}$ and $\bar{G}_{A1}$. For the present work we shall omit the subscript 1 for convenience. Later on we shall mention briefly the characteristics of the function of the second kind.

III. Eigenfunction Expansion of $\bar{G}_A$ for a Rectangular Cavity

The rectangular cavity under consideration has the configuration shown in Fig. 1.

Following the Ohm–Rayleigh method [2], [3] we expand first the singular function $I\delta(\bar{R} - \bar{R}')$ in terms of the vector wave functions $\bar{L}_{00}$, $\bar{M}_{e0}$, and $\bar{N}_{0e}$ defined as follows:

$$\bar{L}_{00} = \nabla\psi_{00} \tag{6}$$

$$\bar{M}_{e0} = \nabla \times (\psi_{e0}\hat{z}) \tag{7}$$

$$\bar{N}_{0e} = \frac{1}{K}\nabla \times \nabla \times (\psi_{0e}\hat{z}) \tag{8}$$

where

$$\psi_{00} = \sin k_x x \sin k_y y \sin k_z z$$

$$\psi_{e0} = \cos k_x x \cos k_y y \sin k_z z$$

$$\psi_{0e} = \sin k_x x \sin k_y y \cos k_z z$$

$$k_x = \frac{m\pi}{a}, \qquad k_y = \frac{n\pi}{b}, \qquad k_z = \frac{l\pi}{c}$$

$$m,n,l = 0,1,2,\cdots$$

$$K^2 = k_x^2 + k_y^2 + k_z^2.$$

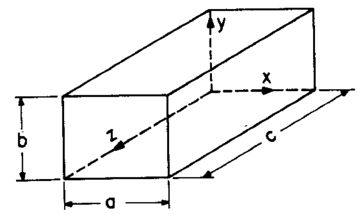

Fig. 1. A rectangular cavity and the designation of the coordinate system.

As a result of the orthogonal properties of these vector wave functions, we find

$$I\delta(\bar{R} - \bar{R}') = \sum_{l,m,n} C_{mn} \left[\frac{k_c^2}{K^2}\bar{L}_{00}\bar{L}_{00}' + \bar{M}_{e0}\bar{M}_{e0}' + \bar{N}_{0e}\bar{N}_{0e}'\right] \tag{9}$$

where the primed functions are defined with respect to the primed variables x', y', and z' pertaining to $\bar{R}'$ and

$$C_{mn} = \frac{4(2 - \delta_0)}{abck_c^2}$$

$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\delta_0 = \begin{cases} 1, & l \text{ or } m \text{ or } n = 0 \\ 0, & l, m, n \neq 0. \end{cases}$$

The constant k_c corresponds to the cutoff wave number of a rectangular waveguide with a cross section $a \times b$. The derivation of (9) follows the same steps as described in [2] and [3]. In view of (1), we find

$$\bar{G}_A = \sum_{l,m,n} \frac{C_{mn}}{K^2 - k^2} \cdot \left[\frac{k_c^2}{K^2}\bar{L}_{00}\bar{L}_{00}' + \bar{M}_{e0}\bar{M}_{e0}' + \bar{N}_{0e}\bar{N}_{0e}'\right]. \tag{10}$$

The series containing the $\bar{L}_{00}\bar{L}_{00}'$ terms is responsible not only for the field in a source region but also contributes to the electric field in a source-free region as will be shown subsequently. The expression for $\bar{G}_A$ as given by (10) can be written in a different form using the modal functions commonly used in waveguide theory, particularly by Felsen and Marcuvitz [4] and their followers. These modal functions have also been used by Morse and Feshbach. They are defined by

$$\bar{l}_0 = \phi_0\hat{z} \tag{11}$$

$$\bar{m}_e = \nabla_t\phi_e \times \hat{z} \tag{12}$$

$$\bar{n}_0 = \nabla_t\phi_0 \tag{13}$$

where

$$\phi_0 = \sin k_x x \sin k_y y$$

$$\phi_e = \cos k_x x \cos k_y y$$

$$k_x = \frac{m\pi}{a}, \qquad k_y = \frac{n\pi}{b}, \qquad m,n = 0,1,\cdots.$$

The vector wave functions which we used in expanding $\bar{G}_A$ are related to these modal functions. Thus it is not difficult to show

$$\bar{L}_{00} = \bar{n}_0 \sin k_z z + k_z \bar{l}_0 \cos k_z z$$
$$\bar{M}_{e0} = \bar{m}_e \sin k_z z$$
$$\bar{N}_{0e} = \frac{1}{K}(-k_z \bar{n}_0 \sin k_z z + k_c^2 \bar{l}_0 \cos k_z z).$$

In terms of $\bar{l}_0$, $\bar{m}_e$, and $\bar{n}_0$, we can write (10) in the form

$$\bar{G}_A = \sum_{l,m,n} \frac{C_{mn}}{K^2 - k^2}[k_c^2 \bar{l}_0 \bar{l}_0' \cos k_z z \cos k_z z' + (\bar{m}_e \bar{m}_e' + \bar{n}_0 \bar{n}_0') \sin k_z z \sin k_z z']. \quad (14)$$

Now the sum over the index l can be evaluated in a closed form by making use of the relations [5]

$$\sum_{l=1} \frac{1}{K^2 - k^2} \sin k_z z \sin k_z z' = \frac{c}{2k_g \sin k_g c} \begin{Bmatrix} \sin k_g(c - z) \sin k_g z' \\ \sin k_g z \sin k_g(c - z') \end{Bmatrix} z \gtrless z' \quad (15)$$

$$\sum_{l=0} \frac{1}{K^2 - k^2} \cos k_z z \cos k_z z' = \frac{-c}{2k_g \sin k_g c} \begin{Bmatrix} \cos k_g(c - z) \cos k_g z' \\ \cos k_g z \cos k_g(c - z') \end{Bmatrix} z \gtrless z' \quad (16)$$

where

$$k_g = (k^2 - k_c^2)^{1/2}.$$

An equivalent expression for (14) is therefore given by

$$\bar{G}_A = \sum_{m,n} C_{mn}^*[k_c^2 \bar{l}_0 \bar{l}_0' g_{mn} + (\bar{m}_e \bar{m}_e' + \bar{n}_0 \bar{n}_0') f_{mn}] \quad (17)$$

where

$$C_{mn}^* = \frac{2(2 - \delta_0)}{abk_c^2 k_g \sin k_g c}$$

$$f_{mn} = \begin{Bmatrix} \sin k_g(c - z) \sin k_g z' \\ \sin k_g z \sin k_g(c - z') \end{Bmatrix} z \gtrless z' \quad (18)$$

$$g_{mn} = \begin{Bmatrix} \cos k_g(c - z) \cos k_g z' \\ \cos k_g z \cos k_g(c - z') \end{Bmatrix} z \gtrless z'. \quad (19)$$

Equation (17) is the same as the one previously given by Morse and Feshbach [1, eq. 13.3.47] based on the method of scattering superposition starting with dyadic Green's function for $\bar{G}_A$ pertaining to a rectangular waveguide.

In principle, once $\bar{G}_A$ is known one can find the vector potential function $\bar{A}$ for any arbitrary current source, including source of the aperture type, as shown by Morse and Feshbach. To find $\bar{E}$, the electric field, another differential operation is needed as

$$\bar{E} = i\omega\left(\bar{A} + \frac{1}{k^2}\nabla\nabla \cdot \bar{A}\right).$$

If we use (17) for $\bar{G}_A$, then the differential operation involves a series with a discontinuous derivative in the source region that must be executed with due care. For this reason, it appears more appealing to deal with $\bar{G}_e$, the dyadic Green's function of the electric type. Once $\bar{G}_e$ is known one can find $\bar{E}$ by applying the formula

$$\bar{E}(\bar{R}) = i\omega\mu_0 \iiint \bar{G}_e(\bar{R} \mid \bar{R}') \cdot \bar{J}(\bar{R}')\, dv' - \oiint \widetilde{\nabla' \times \bar{G}_e(\bar{R}' \mid \bar{R})} \cdot [\hat{n} \times \bar{E}(\bar{R}')]\, ds' \quad (20)$$

where the sign $\sim$ over $\nabla' \times \bar{G}_e$ denotes the transposition of the entire dyadic function. In fact, it is known [2], [3] that

$$\widetilde{\nabla' \times \bar{G}_{e1}(\bar{R}' \mid \bar{R})} = \nabla \times \bar{G}_{e2}(\bar{R} \mid \bar{R}') \quad (21)$$

where $\bar{G}_{e2}$ denotes the dyadic Green's function of the second kind of the electric type which satisfies the same equation as $\bar{G}_{e1}$, the first kind or simply $\bar{G}_e$ in our present designation, but with the boundary condition

$$\hat{n} \times \nabla \times \bar{G}_{e2} = 0.$$

A more precisely annotated expression for (20), therefore, should be

$$\bar{E}(\bar{R}) = i\omega\mu_0 \iiint \bar{G}_{e1}(\bar{R} \mid \bar{R}') \cdot \bar{J}(\bar{R}')\, dv' - \oiint \nabla \times \bar{G}_{e2}(\bar{R} \mid \bar{R}') \cdot [\hat{n} \times \bar{E}(\bar{R}')]\, ds'. \quad (22)$$

Because of the convenience of using $\bar{G}_e$ instead of $\bar{G}_A$, it is desirable to give a detailed derivation of the several alternative representations of $\bar{G}_e$, which is understood to be $\bar{G}_{e1}$ in this paper.

IV. Eigenfunction Expansion of $\bar{G}_e$ for a Rectangular Cavity

By applying the Ohm–Rayleigh method to the equation for $\bar{G}_e$ defined by (2) or by substituting (10) into (3), one finds

$$\bar{G}_e = \sum_{l,m,n} C_{mn}\left[\frac{1}{K^2 - k^2}(\bar{M}_{e0}\bar{M}_{e0}' + \bar{N}_{0e}\bar{N}_{0e}') - \frac{k_c^2}{k^2 K^2}\bar{L}_{00}\bar{L}_{00}'\right] \quad (23)$$

where C_{mn}, $\bar{M}_{e0}$, $\bar{N}_{0e}$, and $\bar{L}_{00}$ have been defined before. It is observed that the coefficient attached to the $\bar{L}_{00}\bar{L}_{00}'$ terms is different from the one associated with $\bar{G}_A$, while the coefficient attached to the $\bar{M}_{e0}\bar{M}_{e0}'$ and $\bar{N}_{0e}\bar{N}_{0e}'$ terms remains unchanged. This is because

$$\nabla \cdot \bar{M}_{e0} = 0$$
$$\nabla \cdot \bar{N}_{0e} = 0$$
$$\nabla\nabla \cdot \bar{L}_{00} = -K^2 \bar{L}_{00}$$

and

$$\frac{k_c^2}{(K^2 - k^2)K^2}\left(1 - \frac{K^2}{k^2}\right) = -\frac{k_c^2}{k^2 K^2}.$$

In terms of the modal functions $\bar{l}_0$, $\bar{m}_e$, and $\bar{n}_0$ as defined by (11)–(13), one can write (23) in the form

$$\bar{G}_e = \sum_{l,m,n} \frac{C_{mn}}{K^2 - k^2} \Big[\bar{m}_e\bar{m}_e' \sin k_z z \sin k_z z' + \frac{k_g^2}{k^2} \bar{n}_0\bar{n}_0' \sin k_z z \sin k_z z' + \frac{k_c^2(k^2 - k_z^2)}{k^2} \bar{l}_0\bar{l}_0' \cos k_z z \cos k_z z' - \frac{k_z k_c^2}{k^2} (\bar{l}_0\bar{n}_0' \cos k_z z \sin k_z z' + \bar{n}_0\bar{l}_0' \sin k_z z \cos k_z z') \Big]. \tag{24}$$

Now the series containing the $\bar{l}_0\bar{l}_0'$ terms has a singular term which can be extracted from the sum. Using (19), one finds that

$$\frac{\partial^2 g_{mn}}{\partial z^2} = -k_g^2 g_{mn} - k_g \sin k_g c \delta(z - z'). \tag{25}$$

The singular term involving $\delta(z - z')$ results from the discontinuity of $\partial g_{mn}/\partial z$. In view of (16) and (19) we have

$$-\sum_{l=1} \frac{k_z^2 \cos k_z z \cos k_z z'}{K^2 - k^2} = \frac{\partial^2}{\partial z^2} \sum_{l=0} \frac{\cos k_z z \cos k_z z'}{K^2 - k^2} = \frac{c}{2k_g \sin k_g c} \frac{\partial^2 g_m}{\partial z^2} = -\frac{c}{2}\delta(z - z') - \frac{k_g c}{2 \sin k_g c} g_{mn} = -\frac{c}{2}\delta(z - z') - \sum_{l=0} \frac{k_g^2}{K^2 - k^2} \cos k_z z \cos k_z z'.$$

Thus (24) can be written in the form

$$\bar{G}_e = -\sum_{m,n} C_{mn} \left(\frac{k_c}{k}\right)^2 \frac{c}{2} \delta(z - z')\bar{l}_0\bar{l}_0' + \sum_{l,m,n} \frac{C_{mn}}{K^2 - k^2} \Big[\bar{m}_e\bar{m}_e' \sin k_z z \sin k_z z' + \frac{k_g^2}{k^2} \bar{n}_0\bar{n}_0' \sin k_z z \sin k_z z' + \frac{k_c^4}{k^2} \bar{l}_0\bar{l}_0' \cos k_z z \cos k_z z' - \frac{k_z k_c^2}{k^2} (\bar{l}_0 \bar{n}_0' \cos k_z z \sin k_z z' + \bar{n}_0\bar{l}_0' \sin k_z z \cos k_z z') \Big]. \tag{26}$$

The double series in (26) is recognized as

$$-\frac{1}{k^2} \hat{z}\hat{z}\delta(\bar{R} - \bar{R}')$$

because

$$\delta(\bar{R} - \bar{R}') = \delta(x - x')\delta(y - y')\delta(z - z') = \sum_{m,n} \frac{4}{ab} \phi_0\phi_0'\delta(z - z') \tag{27}$$

where $\phi_0 = \sin k_x x \sin k_y y$. The triple series can be summed over l using (15) and (16) and the additional relations

$$\sum_{l=1} \frac{k_z \sin k_z z \cos k_z z'}{K^2 - k^2} = \frac{-c}{2k_g \sin k_g c} \frac{\partial g_{mn}}{\partial z} \tag{28}$$

$$\sum_{l=1} \frac{k_z \cos k_z z \sin k_z z'}{K^2 - k^2} = \frac{c}{2k_g \sin k_g c} \frac{\partial f_{mn}}{\partial z}. \tag{29}$$

The final expression for $\bar{G}_e$ after this reduction has the form

$$\bar{G}_e = -\frac{1}{k^2} \hat{z}\hat{z}\delta(\bar{R} - \bar{R}') + \sum_{m,n} C_{mn}^* \Big[(\bar{m}_e\bar{m}_e' + \frac{k_g^2}{k^2} \bar{n}_0\bar{n}_0')f_{mn} + \frac{k_c^4}{k^2} \bar{l}_0\bar{l}_0' g_{mn} + \frac{k_c^2}{k^2} \bar{n}_0\bar{l}_0' \frac{\partial g_{mn}}{\partial z} - \frac{k_c^2}{k^2} \bar{l}_0\bar{n}_0' \frac{\partial f_{mn}}{\partial z} \Big] \tag{30}$$

where

$$C_{mn}^* = \frac{2(2 - \delta_0)}{abk_c^2 k_g \sin k_g c}.$$

The function f_{mn} and g_{mn} are defined by (18) and (19).

It should be remarked that an alternative procedure to obtain (30) is to use the formula

$$\bar{G}_e = \left(I + \frac{1}{k^2}\nabla\nabla\right) \cdot \bar{G}_A$$

with $\bar{G}_A$ given by (17). If this procedure is followed the following relations are needed:

$$\nabla \cdot (\bar{m}_e f_{mn}) = 0$$

$$\nabla \cdot (\bar{n}_0 f_{mn}) = -k_c^2 \phi_0 f_{mn}$$

$$\nabla \cdot (\bar{l}_0 g_{mn}) = \phi_0 \frac{\partial g_{mn}}{\partial z}$$

$$\nabla\nabla \cdot (\bar{n}_0 f_{mn}) = -k_c^2 \left(\bar{n}_0 f_{mn} + \bar{l}_0 \frac{\partial f_{mn}}{\partial z}\right)$$

$$\nabla\nabla \cdot (\bar{l}_0 g_{mn}) = \bar{n}_0 \frac{\partial g_{mn}}{\partial z} + \bar{l}_0 \frac{\partial^2 g_{mn}}{\partial z^2}.$$

From the point of view of waveguide theory, a rectangular cavity can be considered as a waveguide terminated by two conducting walls at the ends. For this reason, it is desirable to identify the significance of (30) based on this approach.

It is recalled [3] that the complete expression of the dyadic Green's function of the electric type for an infinite rectangular waveguide is given by

$$\bar{G} = -\frac{1}{k^2}\hat{z}\hat{z}\delta(\bar{R} - \bar{R}') + \sum_{m,n} \frac{i(2 - \delta_0)}{abk_c^2 k_g} \cdot [\bar{M}(\pm k_g)\bar{M}'(\mp k_g) + \bar{N}(\pm k_g)\bar{N}'(\mp k_g)], \qquad z \gtrless z' \tag{31}$$

where

$$\bar{M}(k_g) = \nabla \times [\phi_0 e^{ik_g z}\hat{z}]$$
$$\bar{N}(k_g) = \frac{1}{k}\nabla \times \nabla \times [\phi_e e^{ik_g z}\hat{z}]$$
$$k_g^2 = k^2 - k_c^2$$
$$k_c^2 = k_x^2 + k_y^2$$
$$\phi_0 = \sin k_x x \sin k_y y$$
$$\phi_e = \cos k_x x \cos k_y y.$$

It should be noted that the term $-[\hat{z}\hat{z}\delta(\bar{R} - \bar{R}')]/k^2$ was missing in the old treatment as found in [2], but amended in [3].

For the cavity, we construct $\bar{G}_e$ by the method of scattering superposition. Thus we let

$$\bar{G}_e = \bar{G} + \sum_{m,n} \frac{i(2 - \delta_0)}{abk_c^2 k_g} [\bar{M}(k_g)\bar{A}_1 + \bar{M}(-k_g)\bar{A}_2 + \bar{N}(k_g)\bar{B}_1 + \bar{N}(-k_g)\bar{B}_2] \tag{32}$$

where the scattering terms represent the reflected TE and TM modes from the two end walls. After applying the boundary condition $\hat{z} \times \bar{G}_e = 0$ at $z = 0$ and $z = c$ we can determine the unknown coefficients $\bar{A}$'s and $\bar{B}$'s. The final result is given by

$$\bar{A}_1 = \frac{-1}{\sin k_g c} \sin k_g(c - z')\bar{m}_e'$$
$$\bar{A}_2 = \frac{-1}{\sin k_g c} e^{ik_g c} \sin k_g z' \bar{m}_e'$$
$$\bar{B}_1 = \frac{-1}{k \sin k_g c} \cdot [k_g \sin k_g(c - z')\bar{n}_0' + k_c^2 \cos k_g(c - z')\bar{l}_0']$$
$$\bar{B}_2 = \frac{ie^{ik_g c}}{k \sin k_g c} [-k_g \sin k_g z' \bar{n}_0' + k_c^2 \cos k_g z' \bar{l}_0]$$

where $\bar{m}_e$, $\bar{n}_0$, and $\bar{l}_0$ denote the modal functions defined previously by (11)–(13). By summing all the parts in (32) it can be shown that the result is identical to (30) as it should be. Furthermore, if we introduce the vector wave functions defined by

$$\bar{M}_{e0}[k_g z] = \nabla \times [\phi_e \sin k_g z\hat{z}]$$
$$\bar{M}_{e0}[k_g(c - z)] = \nabla \times [\phi_e \sin k_g(c - z)\hat{z}]$$
$$\bar{N}_{0e}[k_g z] = \frac{1}{k}\nabla \times \nabla \times [\phi_0 \cos k_g z\hat{z}]$$
$$\bar{N}_{0e}[k_g(c - z)] = \frac{1}{k}\nabla \times \nabla \times [\phi_0 \cos k_g(c - z)\hat{z}].$$

Then (30) can be written in the following compact form:

$$\bar{G}_e = -\frac{\hat{z}\hat{z}\delta(\bar{R} - \bar{R}')}{k^2} + \sum_{m,n} C_{mn}^* \begin{Bmatrix} \bar{M}_{e0}[k_g(c - z)]\bar{M}_{e0}'[k_g z'] \\ \bar{M}_{e0}[k_g z]\bar{M}_{e0}'[k_g(c - z')] \end{Bmatrix} \begin{matrix} - \bar{N}_{0e}[k_g(c - z)]\bar{N}_{0e}'[k_g z'] \\ - \bar{N}_{0e}[k_g z]\bar{N}_{0e}'[k_g(c - z')] \end{matrix}\Bigg\} \quad z \gtrless z' \tag{33}$$

where

$$C_{mn}^* = \frac{2(2 - \delta_0)}{abk_c^2 k_g \sin k_g c}.$$

In summary, we have derived three different but equivalent representations of $\bar{G}_e$ for a rectangular cavity stated by (23), (30), and (33). The functions for cylindrical and spherical cavities in the triple sum form are also available [6]. For cylindrical cavities it is possible to reduce the triple sum into a double sum. This is, however, not possible for spherical cavities.

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